A UTILIZAÇÃO DOS PROGRAMAS MATHCAD E MATLAB COMO FERRAMENTAS DE APRENDIZAGEM NAS DISCIPLINAS DE GEODÉSIA DO CURSO DE ENGENHARIA CARTOGRÁFICA DA UFRGS

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RESUMO

A inclusão dos programas MathCad e MatLab como uma ferramenta de auxílio na aprendizagem das disciplinas de geodésia do curso de Engenharia Cartográfica tem tido uma excelente aceitação parte dos alunos. Esta reação é motivada pelo fato das expressões matemáticas serem implementadas nesses programas exatamente como os alunos às escrevem em seus cadernos. Além disso, estes programas possuem todos os recursos sofisticados de programação, e principalmente não exigem profundos conhecimentos de linguagem de programação. A implementação de soluções, para os problemas abordados em diferentes áreas do conhecimento, utilizando-se desse ferramental, constitui atualmente tema de grande interesse e passaram a evoluir com notável rapidez tanto no aspecto teórico como no prático. Os programas MathCad e MatLab, fornecem uma grande variedade de ferramentas matemáticas que possibilitam um aprimoramento, tanto na forma de aprendizado da disciplina como na familiarização do aluno com a programação computacional. Isso vem possibilitando a solução desses problemas com uma extraordinária rapidez. Este trabalho tem como objetivo mostrar a aplicabilidade da utilização dos programas MathCad e o MatLab no ensino da Geodésia.

Palavras-chave: linguagens de programação, MathCad, MatLab, ferramentas de aprendizagem, Geodésia

USE OF MATHCAD AND MATLAB AS GEODESY LEARNING TOOLS FOR THE CARTOGRAPHIC ENGINEERING COURSE AT THE UFRGS

ABSTRACT

Students have broadly accepted the introduction of MathCad and MatLab as aiding tools for geodesy learning issue in the Cartographic Engineering undergrat course at UFRGS. This is explained by the fact that these programs implement mathematical expressions in the same way students write them out in their notebooks. In addition, these programs feature all the sophisticated resources and, most importantly, they do not require deep knowledge of the programming languages. The implementation of solutions by means of these tools, for the problems addressed in different academic areas, is an issue that has gained great interest and has evolved very quickly, both in their theoretical and practical aspects. MathCad and MatLab, provide a great variety of mathematical tools that permit the improvement, both in the learning of the subjects as well as in the possibility of the student becoming familiar with computer programming. This has led to an outstandingly fast problem solution process. The aim of this paper is to demonstrate the applicability of the use of MathCad and MatLab in geodesy teaching.

Keywords: programming languages, MathCad, MatLab, learning tools, Geodesy

1. INTRODUCTION

A common difficulty that students face in an engineering undergraduate course is related to the building of a computation solution for the problems offered in the disciplines of the course. Although it might appear to be extremely simple, when a programming tool is used, it is necessary to build up thinking in a structured way (working as an algorithm), editing algorithms in a programming language (for example: C, Pascal or Fortran), or to use a development environment for program building (for example: Delphi, Visual Basic or Visual C++). In general, when transmitting information to computer, the knowledge of one of the above languages is necessary, but this is something slow and uninteresting for most students. A way to overcome this problem is to simplify learning through the use of tools such as Mathematica, MathCad or MatLab, which are similar to text editors focusing mathematics learning. With this type of language, the student programming can explore all basic concepts and also visualize the execution of programs, because they are written in a very simple language.

We should highlight that the student-computer interaction needs a mediator to offer a knowledge building process, in other words, an experienced professional in the learning process. A person who understands, in depth, the material the student is working with and the potential capabilities of the computer (VALENTE, 2003). This knowledge needs to be used by the teacher in order to interpret the ideas of the students and also to become appropriately involved in the learning environment, so as to contribute to the knowledge building process of the student. In addition, considering the modifications in the role of errors (no longer to be punished, but to be corrected), VALENTE (2003) states that this approach requires significant changes in the educational system. This paper aims to show the potential of possible aplication and implications that Mathcad and Matlab can offer to geodesy teaching, for the cartographic engineering course of UFRGS.

2. PROGRAM DESCRIPTION

Both MathCad and MatLab are presented as high-level programming softwares that are very similar. In terms of popularity, MatLab has gained more recognition than MathCad. According to the opinions of some critics, MatLab is seen as the best program in the area, something which to other program can challenge. Of course there are other preferences, for example: Mathematica (LIMA & LIMA, 2003), which is extensively used in the disciplines taught at the Mechanical Engineering course at UFPB. Therefore, the three programs, i.e. Mathematica, MatLab and MathCad, each have strong assets as well as their weak points, and if one is remarkable in one aspect or another, the other two may not be so good in that specific aspect. The Federal University of Rio Grande do Sul, MatLab is favored, to such an extent that there is an institutional copy of the program available at the University. In summary, both MatLab and MathCad are tools for excellent mathematic use, and it can be applied to the different areas of knowledge.

2.1 MATHCAD

MathCad is a technical tool for calculations, focusing on standard math notation. It is a technical text processor, which permits mathematic calculations to integrate to the text. The software interface is similar to a sheet of paper, on which mathematical expressions can be written. The program was developed by *MathSoft, Inc.* and provides a great capacity for numeric and analytical calculations as well as graphing capabilities. It offers tools for the publication of results on the Internet, through the use of Microsoft's Internet Explorer. A page on the Internet named <u>MathCAD Collaboratory</u>, contains several contributions from program users ([01], [02], NITZ & GALHA, 2003).

2.2 MATLAB

Cleve Moler, from the Department of Computing Sciences at the University of New Mexico, USA. developed MatLab in the early of the 80's. It is a high-performance interactive "software" used for numeric calculations. A includes numeric analysis, calculation with matrices, symbol processing and graph building within a relatively easy environment. Problems and solutions are expressed as they are mathematically being written out, contrary to the traditional programming. MatLab is an interactive system, with a basic information element, the matrix, which requires no dimensioning. The system permits the resolution of many numerical problems in just a fraction of the time that we would need to write a similar program in Fortran, Pascal or C languages. In addition, the solutions to the problems are expressed almost exactly as they are mathematically written. (MATLAB, 1991; HANSELMAN & LITTLERFIELD, 2003).

3. APPLICATIONS IN GEODESY

The uses of MathCad and MatLab are unlimited, not only for the teaching of Geodetic Sciences but also applicable to any Cartographic Engineering course. With such programs we may carry out activities in the areas of Photogrametry, Observation Adjusting (STRANG & BORREM, 1997) and Geodesy (LEICK, 2004). The Cartographic Engineering course at UFRGS teaches of Geodesy in three compulsory complementary disciplines. Geodesy I, addresses issues such as Geometrical Geodesy, Physical Geodesy and Geodesy III, focus on the aspects related to Spatial Geodesy. The exercises that are developed in class can be easily resolved with the use of these programs. In order to provide an example of these programs, applied

to Geodesy disciplines, some practical examples on the use of MathCad and MatLab in the classroom are addressed in the following paragraphs.

3.1 GEOMETRICAL GEODESY

The aims of Geodesy are the positioning points and the portraying of fields and gravity, including the temporal variations of both. The complexity of Geometry and the distribution of the Earth mass led to the use of a simplified model, the ellipsoid. Geometrical Geodesy aims to describe the operations performed on this surface. As a practical exercise to be done in class, we provide, as an example, the application of these programs, the transformation of geocentric cartesian coordinates to geodetic ones and vice-versa. The transformation of geocentric cartesian coordinates into local geodetic coordinates and vice-versa can be obtained, in a relatively simple way, from the parameters of the ellipsoid, from transformation constants and the already known coordinates, as shown in Fig. 1, regarding the implementation of these transformations with MathCad.

 $geod_xyz(\phi, \lambda, h) := \begin{bmatrix} \text{"-TRANSFORMATION OF GEODESIC COORDENATES TO CARTESIAN COORDENATES (MATHCAD)"} \\ \text{"Ellipsoid Parameters: SAD69 (South American Datum)"} \\ a_e \leftarrow (6378137) \\ b_e \in (6356752.314) \\ e \leftarrow (6356752.314) \\ e \leftarrow (1 - e^2 - b_e e^2)^{0.5} \\ e \leftarrow trunc(\phi) + \frac{trunc[(\phi - trunc(\phi)) \cdot 100]}{60} + \frac{[[(\phi \cdot 100 - trunc(\phi \cdot 100)) \cdot 100]]}{3600} \\ \lambda \leftarrow trunc(\lambda) + \frac{trunc[(\lambda - trunc(\lambda)) \cdot 100]}{60} + \frac{[[(\lambda \cdot 100 - trunc(\lambda \cdot 100)) \cdot 100]]}{3600} \\ \lambda \leftarrow trunc(\lambda) + \frac{trunc[(\lambda - trunc(\lambda)) \cdot 100]}{60} + \frac{[[(\lambda \cdot 100 - trunc(\lambda \cdot 100)) \cdot 100]]}{3600} \\ N \leftarrow \frac{a_e}{(1 - e^2 \cdot sin(\phi \cdot deg)^2)^{0.5}} \\ X \leftarrow (N + h) \cdot cos(\phi \cdot deg) \cdot cos(\lambda \cdot deg) \\ Y \leftarrow (N + h) \cdot cos(\phi \cdot deg) \cdot sin(\lambda \cdot deg) \\ Z \leftarrow [[(1 - e^2) \cdot N + h] \cdot sin(\phi \cdot deg)] \\ Z \leftarrow [(1 - e^2) \cdot N + h] \cdot sin(\phi \cdot deg) \\ Z \leftarrow [(X + h) \cdot cos(\phi \cdot deg) \cdot sin(\lambda \cdot deg)] \\ Z \leftarrow [(X + h) \cdot sin(\phi \cdot deg)] \\ = EIC (1 - TRANSFORMATION OF CEODETIC COORDINATES TO CARTESIAN COORD$

FIG. 1 – TRANSFORMATION OF GEODETIC COORDINATES TO CARTESIAN COORDINATES (MATHCAD)

Fig. 1 shows the development of the coordenate transformation problem with MathCad, which can also be easily implemented in the MatLab environment (Fig. 2). The terms used are to be found in MONICO (2000). It is obvious that in this example, the implementation process is far simpler, all the student has to do is introduce the terms as they are found.

Attention should be given to the fact that latitude and longitude values have to be transformed into radians. In the case of latitude value (in degrees minutes and seconds), the implementation of a function for the conversion in decimal degrees becomes necessary, as seen on Figures 1 and 2. The result is perceived immediately, and for that reason the correction of errors committed during the data loading process can be easily detected.

MATLAB Editor/Debugger - [GEOD_XYZ.M - D:\ATIVIDADES_DIDATICAS\DISCIPLINAS - 25EM2002\GEODI	SIA I\p
function geoxyz(fi,lambda,h) format long g	ŕ
a=6378137.000; b=6356752.314;	
<pre>fi=fi*(-1); fi=floor(fi)+floor((fi-floor(fi))*100)/60+(((fi*100-floor(fi*100))*100))/3600; fi=fi*(-1); fi=i*pi/180;</pre>	
lambda=lambda*(-1); lambda=floor(lambda)+floor((lambda-floor(lambda))*100)/60+(((lambda*100-floor(l lambda=lambda*(-1);	ambda*100))
lambda=lambda*pi/180;	
e2=(a^2-b^2)/(a^2); aux=(e2*sin(fi)^2); N=a/((1-aux)^0.5);	
X=(N+h)*cos(fi)*cos(lambda)	
Y=(N+h)*cos(fi)*sin(lambda)	
Z=(((1-e2)*N)+h)*sin(fi)	
x	Ľ
GEOD_XYZ	
Ready Line 21	9:03 AM

FIG. 2 – TRANSFORMATION OF GEODETIC COORDINATES IN CARTESIAN COORDINATES (MATLAB)

The expressions implemented the transformation of geodetic coordinates in geocentric Cartesian ones. However, the user is generally more interested in the inverse transformation, that is, in obtaining the geodetic coordinates starting from the Cartesian ones (MONICO, 2000).

Although this transformation can be obtained through direct expression, it is usually done in the iterative way. Similarly to the previous case, the implementation was made in the MathCad environment (Fig. 3).

.YZ_GEOD(X,Y,Z) :=	"TRANSFORMATION OF CARTESIAN COORDENATES TO GEODESIC COORDENATES"
	"Ellipsoid Parameters: SAD69 (South American Datum) "
	a_e ← 6378137
	b_e ← 6356752.314
	$\mathbf{e} \leftarrow \frac{\left(\mathbf{a}_{\underline{e}}\mathbf{c}^2 - \mathbf{b}_{\underline{e}}\mathbf{c}^2\right)^{0.5}}{\mathbf{a}_{\underline{e}}\mathbf{c}}$
	" LONGITUDE"
	$tan \lambda \leftarrow \frac{Y}{X}$
	$\lambda_g \leftarrow \left[\left(\operatorname{atan}(\operatorname{tan}\lambda) \right) \frac{180}{\pi} \right]^{(-1)}$
	$\lambda_{gms} \leftarrow (floor(\lambda_{g})) + \left[\frac{floor(\lambda_{g} - trunc(\lambda_{g})) \cdot 6i}{100}\right] + \left[\left[(\lambda_{g} - trunc(\lambda_{g})) \cdot 0.6 - \frac{floor(\lambda_{g} - trunc(\lambda_{g})) \cdot 6i}{100}\right] \cdot 0.6\right]$
	$\lambda_gms \leftarrow (\lambda_gms) - 1$
	- LATITUDE"
	$p \leftarrow (X^2 + Y^2)^{\circ \circ \circ}$
	$tan\phi \leftarrow \frac{Z}{p \cdot (1 - e^2)}$
	$\phi o \leftarrow atan(tan\phi o)$
	$\phi_a \leftarrow \phi_1 \leftarrow 0$
	$\phi_2 \leftarrow \phi_0$
	while $ \phi_2 - \phi_1 > 0.0000000000000000000000000000000000$
	$\phi_1 \leftarrow \phi_2$
	$vo \leftarrow \frac{a_c}{\left(1 - c^2 \sin(\phi_c 1)^2\right)^{0.5}}$
	$ho \leftarrow \frac{p}{\cos(\phi_{\perp}I)} - vo$
	$ \tan\phi \leftarrow \frac{Z}{p\left[1-c^2\left(\frac{vo}{v}\right)\right]} $
	$\left[\frac{1}{\sqrt{1+1}} \frac{1}{\sqrt{1+1}} \frac{1}{\sqrt{1+1}} \right]$
	$\phi_2 \leftarrow \operatorname{atan}(\operatorname{tan}\phi)$
	$\phi_a \leftarrow \phi_a \cdot \frac{^{180}}{\pi}$
	$\phi_{gms} \leftarrow (floor(\phi_{a})) + \frac{floof(\phi_{a}a - trunc(\phi_{a})) \cdot s0}{100} + \left[(\phi_{a}a - trunc(\phi_{a})) \cdot 0.6 - \frac{floof(\phi_{a}a - trunc(\phi_{a})) \cdot s0}{100} \right] 0.6 - \frac{floof(\phi_{a}a - trunc(\phi_{a})) \cdot s0}{100} = 0.6 - \frac{floof(\phi_{a}a - trunc(\phi_{a})) \cdot s0}{$
	$\phi_{gms} \leftarrow \phi_{gms}(-1)$
	¢_gms
	" ALTITUDE"
	$v \leftarrow \frac{a_e}{\left(1 - e^2 \sin(\phi_{ea} \cdot deg)^2\right)^{0.5}}$
	$h \leftarrow \left(\frac{p}{\cos(\phi_{\perp} a \cdot deg)}\right) - v$
	(ϕ_{gms})
	λ_gms
	I h J

FIG. 3 - TRANSFORMATION OF CARTESIAN COORDINATES TO GEODETIC COORDINATES

Another typical problem in Geometrical Geodesy refers to the transport of coordinates. The vertices of triangulations, polygonizations or trilateralizations are materialized through frames and their coordinates (latitude and longitude) are mathematically determined. The basic problem is the transporting of coordinates from an initial point to all the remaining vertices. This is normally done on the referential ellipsoid, with the terms developed by surface geometry. One of the issues, proposed in the discipline, is the calculation to transport geographical coordinates and UTM. Here, have an example of a direct problem, using geographical coordinates. The direct problem of the transport of coordinates is this: with the geodetic coordinates (latitude and longitude) of a given point, them the ellipsoidal distance to the following point, and the azimuth of the side which liaises them, coordinates of the second point and the counter-azimuth have to be determined. This type of problem can be easily implemented in MathCad and MatLab languages, as seen in Fig. 4.



Contrary to Pascal, Fortran and C languages, which demand a number of procedures for the imple-

mentation of this example, such as the declaration on variables, entry and existing routines as well as the calculation itself, none of these are needed when using MathCad and MatLab.

3.2 PHYSICAL GEODESY

Physical Geodesy addresses the study of gravity and its geodetic applications. The information on the field of Earth gravity is used in the studies on the form, structure and dynamic processes taking place within the planet.

Specifically to this subject, both MatLab and MathCad have been used for the visualization of spheric harmonics, in the reduction of gravimetric observations and in the calculations of the geoidal model. A practical example can be seen on Fig. 5, where the function $plot_ssh(n, m, t)$, developed at MatLab, is used to visualize spheric harmonics.



FIG. 5 – EXAMPLE OF AN MATLAB APPLICATION IN PHYSICAL GEODESY

Fig. 6 shows the procedure for the use of the function developed and the graphic representation of the spheric harmonic.



FIG. 6 - VISUALIZATION OF SPHERICAL HARMONICS

Fig. 6 exemplifies the spherical harmonic degree and zero-order, which represents the sphere. But the student is completely free to alternate the degree and the order of the spherical degree, so that the student will be able to visualize the changes taking place with these modifications. In this function, the possibility exists to modify the resolution of the visualization of the spherical harmonic. For this reason, a third element in the function $plot_ssh(n, m, t)$ was implemented, where t is the number of times the surface will be evaluated, on the interval of each parametric variable. The higher the t value is, the more detailed the surface will be, but the problem is that the process as well as the figure visualization get slower. The vectors of the parameters n and m, which are responsible for the degree and the order of the spheric harmonics can be replaced. The two vectors should be of the same size, and the resulting harmonic is the sum of the harmonics specified in each line of the two vectors.

3.3 SPATIAL GEODESY

The use of programs such as MathCad and MatLab in Spatial Geodesy is challenging. There are countless applications, and we may cite here some studies developed in MathCad and MatLab; for example, on the satellite positioning, (WITCHAYANGKOON, 2000), in the determination of parameters between referential systems, (CASTRO ET. AL.,2001), and in network adjustment GPS (LEICK, 2004).

A routine was developed in MathCad for determining parameters between geodetic systems to demonstrate the potential of the use of these programs in Spatial Geodesy (Fig. 7). In order to resolve this problem, we need to know the cartesian coordinates of points in two different geodetic systems, taking as an example the work by CASTRO et al. (2001), where the determination of the parameters between SAD69 (South American Datum 1969) and SIRGAS (South American Geocentric Reference System) were evaluated. In order to determine the transformation between two cartesian systems, the definition of seven parameters becomes necessary, i.e.: 3 translations, 3 rotations and a scale factor. The estimate of the parameters is made by the application of the adjustment through the least square method, using the parametric method (CASTRO et al.,2001). In this case, to implement this problem in MathCad, some operations are required, which are not available under the form of the function. For that purpose, MathCad includes a programming language which permits the creation of a mechanism and of special calculation functions by means of the use of programming structures, such as: conditional operators, defect treatment operators, iteractive structures, among others. (See Fig. 7).

The use of MathCad and MatLab in the Cartographic Engineering course was firstly introduced in Geodesy and Observation Adjustment. These new learning tools started to be used thanks to the initiative of the students, who at present, use these tools to solve the exercise lists, distributed throughout course, in disciplines such as of Observation Adjustment, Photogrametry, Cartography and Geodesy.

4. CONCLUSIONS

The use of the computer as a transformation tool and as a learning aid in universities, even before the challenges portrayed by this transformation, is a promising and smart one. The use of programs such as MathCad and MatLab, appears to be an excellent tool, not only to assist in the students' performance to solve problems, posed by different disciplines, but as a simplified manner to understand computing procedures, which are much more complicated in other languages. The infromation technology initiatives as to the use of learning process tools, similar to those focused in this paper, are being dissiminated throughout Brazil. (LIMA & LIMA, 2003; D'AMBROSIO, 2003).

The way of teaching with these new tools will be a revolution if we simultaneously modify the conventional paradigms of the teaching process, which keep teachers and students apart. With these new tools we will be able to provide the learning process an upgrade, without affecting the core issues. The use of programs, for this purpose, is a new communication tool, which may become instrumental to review, enhance and modify many of the current teaching and learning procedures.

```
Rp(n,Q,Z) := N \leftarrow (1 \ 0 \ 0 \ 0 \ 0 \ 0)^T
                             mz \leftarrow rows(Q) - 1
                              \quad \text{for} \quad r\in\,0\,..\,\ n
                                 S \leftarrow N_0
                                    rx \leftarrow N_1
                                    ry \leftarrow N_2
                                   rz \leftarrow N_2
                                   Tx \leftarrow N_A
                                   Ty \leftarrow N_{2}
                                   Tz \leftarrow N_6
                                   m11 \leftarrow \cos(ry) \cdot \cos(rz)
                                  m12 \leftarrow sin(rx) \cdot sin(ry) \cdot cos(rz) + cos(rx) \cdot sin(rz)
                                   m13 \leftarrow -cos(rx) \cdot sin(ry) \cdot cos(rz) + sin(rx) \cdot sin(rz)
                                   m21 \leftarrow -cos(ry) \cdot sin(rz)
                                   m22 \leftarrow -\sin(rx) \cdot \sin(ry) \cdot \sin(rz) + \cos(rx) \cdot \cos(rz)
                                   m23 \leftarrow \cos(rx) \cdot \sin(ry) \cdot \sin(rz) + \sin(rx) \cdot \cos(rz)
                                   m31 \leftarrow sin(ry)
                                   m32 \leftarrow -sin(rx) \cdot cos(ry)
                                    m33 \leftarrow \cos(rx) \cdot \cos(ry)
                                    for \ i \in 0 \, .. \ mz
                                        x \leftarrow Q_{i,0}
                                          y \leftarrow Q_{i,1}
                                         z \leftarrow Q_{i,2}
                                          XS \leftarrow m11 \cdot x + m21 \cdot y + m31 \cdot z
                                          Xf \leftarrow S \cdot (-sin(ry) \cdot cos(rz) \cdot x + sin(ry) \cdot sin(rz) \cdot y + cos(ry) \cdot z)
                                          Xk \leftarrow S \cdot (m21 \cdot x - m11 \cdot y)
                                          \text{YS} \leftarrow \text{m12} \cdot \text{x} + \text{m22} \cdot \text{y} + \text{m32} \cdot \text{z}
                                           Yw \leftarrow -S \cdot (m13 \cdot x + m23 \cdot y + m33 \cdot z)
                                          Yf \leftarrow S \cdot (\sin(rx) \cdot \cos(ry) \cdot \cos(rz) \cdot x - \sin(rx) \cdot \cos(ry) \cdot \sin(rz) \cdot y + \sin(rx) \cdot \sin(ry) \cdot z)
                                          Yk \leftarrow S \cdot (m22 \cdot x - m12 \cdot y)
                                          ZS \ \leftarrow \ m13 \cdot x \ + \ m23 \cdot y \ + \ m33 \cdot z
                                          Zw \leftarrow -S \cdot (m12 \cdot x + m22 \cdot y + m32 \cdot z)
                                          Zf \leftarrow S \cdot (-cos(rx) \cdot cos(ry) \cdot cos(rz) \cdot x + cos(rx) \cdot sin(ry) \cdot y + cos(rx) \cdot cos(ry) \cdot sin(rz) \cdot z)
                                         Zk \leftarrow S \cdot (m23 \cdot x - m13 \cdot y)
                                                    (XS 0 Xf Xk 1 0 0)
                                          R_i \leftarrow | YS Yw Yf Yk 0 1 0
                                                    ZS Zw Zf Zk 0 0 1
                                   J \leftarrow R_0
                                   for \ i \in 1 \ .. \ mz
                                    J \leftarrow stack(J, R_i)
                                   \text{for } z \in 0 .. \ mz
                                       UU \leftarrow Z_{z,0} - \left[S \cdot \left(m11 \cdot Q_{z,0} + m21 \cdot Q_{z,1} + m31 \cdot Q_{z,2}\right) + Tx\right]
                                        VV \leftarrow Z_{z,1} - \left[S \cdot \left(m12 \cdot Q_{z,0} + m22 \cdot Q_{z,1} + m32 \cdot Q_{z,2}\right) + Ty\right]
                                        GG \leftarrow Z_{z,2} - \left[S \cdot \left(m13 \cdot Q_{z,0} + m23 \cdot Q_{z,1} + m33 \cdot Q_{z,2}\right) + Tz\right]
                                                    (UU)
                                     U_z \leftarrow \begin{bmatrix} VV \\ GG \end{bmatrix}
                                                     (GG)
                                  K \leftarrow U_0
                                    \text{for } i \in 1 \, .. \, \, mz
                                    K \leftarrow \text{stack}(K, U_i)
                                   Q2 \leftarrow inv((J)^T \cdot J)
                                   X \leftarrow Q2 \cdot (J)^T \cdot K
                                 \mathbf{N} \leftarrow \mathbf{N} + \mathbf{X}
```

FIG. 7 – APPLICATION IN MATHCAD TO DETERMINE PARAMETERS BETWEEN GEODETIC SYSTEMS (ADAPTED FROM [03])

5. **BIBLIOGRAPHY**

[01]http://www.ime.usp.br/~jstern/introdcomp/matman.htm. Acesso em 10/03/2003.

[02]<u>http://www.del.ufms.br/tutoriais/matlab/apresentacao.htm#matlab</u>. Acesso em 10/03/2003.

[03]http:// ottmarlabonde. de / L1 / Helmert -Parameter-Berechnung3.htm. Acesso em 10/03/2003.

- CASTRO, A. L. P., PEREZ, J. A. S., MARINI, M.C., ESPINHOSA, M. L., MÔNICO, J. F. G., CAMARGO, P.
 O. ESTIMATIVA DE PARÂMETROS DE TRANSFORMAÇÃO ENTRE SISTEMAS GEODÉSICOS: Estudo de caso SAD69 e SIRGAS. Série em Ciências Geodésicas: 30 anos de Pós-Graduação em Ciências Geodésicas no Brasil. Curitiba. Imprensa Universitária da UFPR, vol. 1. p.62-81. 2001.
- D'AMBROSIO U. **INFORMÁTICA, CIÊNCIAS e MATEMÁTICA**. <u>http:// www.proinfo.gov.br/</u> <u>biblioteca/textos/txtubiratan.pdf</u>. Acesso em 10/03/2003
- HANSELMAN, D. LITTLEFIELD, B. MATLAB 6 Complete Course. Translation by Claudia Sant'Ana Martins. Sao Paulo: Prentice Hall, 2003.

LEICK, A. GPS SATELLITE SURVEYING, 3rd Edition, 2004, 464 p.

LIMA, L. A.; LIMA, A. G. B. O USO DO SOFTWARE MATHEMATICA[®] NOS CURSOS DE GRADUAÇÃO EM ENGENHARIA MECÂNICA. <u>http:// www.asee.org/ international/INTERTECH2002/517.pdf</u>. Acesso em 10/03/2003.

MATLAB for Windows User's Guide, The Math Works Inc., 1991.

MÔNICO, J. F. G. POSICIONAMENTO PELO NAVSTAR-GPS: descrição, fundamentos e aplicações. Sao Paulo/SP.: Unesp. 2000. 287p.

NITZ, M., GALHA, R. CALCULE COM O MATHCAD. Sao Paulo: Editora Érica, 2003.

- STRANG, G. BORRE, K. 1997. Linear Algebra, Geodesy, and GPS. Wellesley: Wellesley-Cambridge Press. 1997.
- VALENTE, J. A. **O USO INTELIGENTE DO COMPUTADOR NA EDUCAÇÃO**. <u>http://www.Proinfo.gov.br/biblioteca/textos/txtusointe.pdf</u>. Acesso em 30/10/2003.
- WITCHAYANGKOON, B. ELEMENTS OF GPS PRECISE POINT POSITIONING, Thesis, The University of Maine,December,2000.<u>http://www.spatial.maine.edu/Publications/PhD_THESIS/Witchayangkoon2000.pdf</u>

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